▸ STEPHEN MACKERETH, *How constructive is Gödel's Dialectica translation?*.
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Gödel intended for his Dialectica translation to have not only mathematical, but also philosophical and foundational significance. In his 1941 Yale lecture, Gödel argued that his translation gives a *clear, constructive meaning* to the basic notions of intuitionistic logic in their application to arithmetic, clearer and more constructive than the intuitionists' own story [1]. In later writings Gödel shifted his focus, claiming that the translation provides a highly evident *constructive consistency proof* for arithmetic [2, 3].

Gödel has been accused of *reasoning in a circle* in his account of the philosophical significance of his translation: most forcefully by Tait [5], but equally by Troelstra [6]. In Gödel's view, a strictly constructive theory must be essentially quantifier-free; it cannot contain existential quantifiers, nor can it contain propositional operations applied to universal quantifiers. The philosophical significance of the Dialectica translation lies in reducing *quantificational* theories of arithmetic to the *quantifier-free* theory T of primitive recursive functionals of finite type (or "computable functions of finite type," in Gödel's terminology). However, Tait and Troelstra claim that T is not really quantifier-free, because quantificational logic is secretly presupposed by Gödel's definition of "computable function of finite type." If you just unpack the definition, the quantificational logic allegedly reappears.

Curiously, Gödel was *well aware* of the appearance of circularity, yet he repeatedly denied all charges [4, p. 211]. He tried to address the circularity objection directly in footnote **h** of the 1972 version of the Dialectica paper. However, readers have found this footnote to be obscure. I offer a new interpretation of footnote **h**, vindicating Gödel. On my reading, Gödel introduces a new modality $\mathrm{Red}(p)$, read as "p is *reductively provable*." Intuitively, $\mathrm{Red}(p)$ means that p is provable simply by unwinding the chain of definitions of concepts occurring in p, with only minimal supplementation. This concept of reductive provability is closely related to Leibniz's notion of analyticity. I give a precise definition of reductive provability, and I argue that this notion is both *factive* (i.e., we have the axiom $\mathrm{Red}(p) \supset p$) and *decidable*. This is enough to eliminate the quantificational logic from T, meeting the circularity objection. However, there remains a different kind of circularity or impredicativity, which lies in the fact that axioms about reductive provability may occur within reductive proofs.

[1] Kurt Gödel, *In what sense is intuitionistic logic constructive?*, ***Collected Works, Volume III: Unpublished Essays and Lectures*** (Feferman et al., editors), Oxford University Press, New York, 1995, pp. 189–200.

[2] Kurt Gödel, *Über eine bisher noch nicht benützte Erweiterung des finiten Standpunktes*, ***Dialectica***, vol. 12 (1958), pp. 280–287.

[3] Kurt Gödel, *On an extension of finitary mathematics which has not yet been used*, ***Collected Works, Volume II: Publications 1938–1974*** (Feferman et al., editors), Oxford University Press, New York, 1990, pp. 189–200.

[4] Kurt Gödel, Solomon Feferman et al. ***Collected Works, Volume V: Correspondence H–Z***, Oxford University Press, New York, 2003.

[5] William W. Tait, *Gödel's interpretation of intuitionism*, ***Philosophia Mathematica***, vol. 14 (2006), no. 2, pp. 208–228.

[6] Anne Troelstra, *Introductory note to 1958 and 1972*, ***Kurt Gödel: Collected Works, Volume II: Publications 1938–1974*** (Feferman et al., editors), Oxford University Press, New York, 1990, pp. 217–241.