Hyperations for WPO dilators (and the lack thereof)

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Hyperations have been introduced in [1] as a way to transfinitely iterate normal, i.e., strictly increasing continuous, functions on ordinals, refining the notion of Veblen functions. In my master thesis[4], I have investigated the existence of hyperations in the light of reverse mathematics. My main result was that, over ACA_0, this principle is equivalent to

$$\Pi^1_3\text{-}\omega\,\mathrm{RFN}\left(\Pi^1_1\text{-}\,\mathrm{BI}\right),$$

where Π^1_3-$\omega\,\mathrm{RFN}$ denotes reflection over ω-models for Π^1_3-formulas. The formulation of this principle in second order arithmetic employs the notion of dilators introduced by Girard. These are particularly uniform transformations of linear orders, preserving well-foundedness. A categorical description of the Veblen hierarchy as a dilator has been given by Girard in [3].

A variant of dilators for partial orders, called WPO dilators, has been introduced in [2], where they were used to study a functorial version of Kruskal's Theorem. This has led to a rich theory, establishing an equivalence with $\Pi^1_1 - \mathrm{CA}_0$.

In this talk, I want to comment on the interaction between hyperations and WPO dilators. While the well-foundedness proof carries over from the linear setting, the order-theoretic requirement of normality is, unfortunately, rather restrictive in the setting of partial orders. Indeed, I will give an example, showing that a large class of prominent normal WPO dilators, including Higman's order, do not have hyperations that preserve well-partial-orders.

References

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[3] Jean-Yves Girard and Jacqueline Vauzeilles. Functors and ordinal notations. i: A functorial construction of the veblen hierarchy. *Journal of Symbolic Logic*, 49(3):713–729, 1984.

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