Possible world semantics and the contingency of logic

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In modal semantics, when speaking of possible worlds, there seems to be the tacit assumption that logical reasoning will stay constant throughout. That is to say that a logical reasoning valid at one world is valid in all worlds, hence necessary. But what happens then if we decide to consider a possible worlds semantics where different worlds may respond to different logics? What then becomes necessary?

In this talk we expand the possible world semantics for modal logics by not assuming one 'type' of possible worlds in a model, but by considering that different possible worlds might reason under different logics. We focus ourselves on a setting where we combine classical and intuitionistic worlds.

We use $\vdash_i$ to denote pure propositional intuitionistic reasoning even if the language contains $\Box$. In that sense, formulas of the form $\Box A$ behave as propositional variables as far as $\vdash_i$ is concerned. Likewise we consider the $\vdash_c$ relation for classical reasoning. We define so-called *mixed models* which are tuples $\langle W, R, \{l_w\}_{w\in W}, \{T_w\}_{w\in W}\rangle$, where $l_w \in \{i, c\}$ and T_w a set of modal formulas such that

1. $\bot \notin T_w$
2. $T_w \vdash_{l_w} \varphi \Rightarrow \varphi \in T_w$
3. $\Box\varphi \in T_w \iff \forall v(wRv \Rightarrow T_v \vdash_{l_v} \varphi)$
4. $\neg\Box\varphi \in T_w \iff \exists u(wRu \wedge T_u \vdash_{l_u} \neg\varphi)$

We prove soundness of the intuitionistic normal modal logic $\mathsf{iK} + (\mathsf{bem})$ wrt mixed models, where bem is short for 'Box Excluded Middle' and denotes the axiom

$$\Box A \vee \neg\Box A.$$

The logic iK has well-studied birelational semantics with an R relation for the $\Box$ and $\leq$ for intuitionistic implication [1]. We show the correspondence between (bem) and the birelational model frame condition

$$w \leq v \Rightarrow \forall z(wRz \Rightarrow vRz),$$

and we prove soundness and completeness for $\mathsf{iK} + (\mathsf{bem})$ wrt this birelational semantics. We conclude by a proofsketch of completeness for $\mathsf{iK} + (\mathsf{bem})$ wrt mixed models.

These results pave the way for new semantic constructions of Kripke models, raising intriguing mathematical and philosophical questions. It invites us to consider the implementation of more logics, possibly non-comparable, in this construction.

References

[1] M. Božić and K. Došen. Models for normal intuitionistic modal logics. *Studia Logica*, 43(3):217–245, Sep 1984.