

Robert Feys's on Modal Logic

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In a series of influential papers and books published in French or Dutch from the years 1920 to 1965, R. Feys introduced the new developments in logic and presented his own conceptions, particularly in modal logic. Some of these papers have been translated and commented in a book by Max Cresswell and this author, “*Modal Logic in Europe, 1930-1958*”, to appear later this year. In a subsequent book, other works of Feys's and his colleagues with whom he had created the CNRL/NCNL in 1950, the Belgian research center in logic, as well as its journal, “*Logique et Analyse*”.

While admitting the usual restricted sense of modal logic as logic of possibility and necessity, Feys mainly considered modal logic in an extended sense including intuitionistic and multivalued logics. The main problem at the time was that of interpreting these logics and of finding them some applications.

Among interpretations, we summarize the noteworthy contributions of A. Bayart who provided a first possible worlds semantics for first and second order modal logic S5 as well as a completeness proof using Henkin technique for 1st order S5.

Among application and outcome, we consider Feys's suggestion to N. Belnap who had come to work with him in 1958 to provide a Gentzen formulation of W. Ackermann's 1956 “*Strenge Implikation*” theory. Thereafter, Belnap's thesis and his early work with A.R. Anderson were the beginning of the field of Entailment, the “logic of relevance and necessity”.

In this context, we consider R. Meyer's relevant arithmetic, $R^\#$, i.e., Peano arithmetic with the first-order relevant system R (classical logic minus the paradoxical EFQ axiom) as underlying logic.

Although H. Friedman had found a counterexample, there were still ways out. Meyer's original intuition that some elementary proof of FLT must exist could still be proved. At least, it is programmatic and work in progress. This is an intuition also shared by H. Friedman in his conjecture of FLT provable in EFA.

References:

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