



## Robert Feys on Modal Logic

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Max CRESSWELL and Jacques RICHE

## Modal Logic in Europe

1930-1958

## Robert Feys (Mechelen 1889–Herent 1961): CV.

- ▶ Mechelen, Atheneum; 1906 Leuven, ISP HIW, Philosophy.
- ▶ Laboratory of Experimental Psychology (Michotte, Wundt)
- ▶ Mechelen, Theology, 1909; Priest 1912.
- ▶ Jan 1913–Aug 1914, prof Saint Louis Faculties, Brussels.
- ▶ WWI, 10 Aug 14–10 Mar 19, Military Chaplain, 4.5y. on the Ijzer front line; Jul–Sep 1919, Cpt Garrison & Hospitals, Paris.
- ▶ 1919, Secondary school, Nijvel. French & foreign literature!?
- ▶ 1924-1944, College & Faculties St Louis Bsl.
- ▶ 1928, triennial course on Logistique, ISP Leuven.
- ▶ 1944–1960, Full time ISP 1 Logique, 1 Logistique sup

## Robert Feys: publications – main Articles

- ▶ BA, Etude expérimentale sur la conscience des relations, 1908
- ▶ PhD, idem sur la conscience des relations physiques, 1909, lost
- ▶ Rev. of A. I. Shearman, The Development of symbolic Logic, RPL 1912. Cfr C. I. Lewis and L. Couturat
- ▶ La transcription logistique du raisonnement, RPL 1924
- ▶ Le raisonnement en termes de faits dans la logistique russellienne, RPL 1927, 1928
- ▶ Notions/Principes de Logistique, Bxl, St-L, 1936, 37, 39?
- ▶ Les logiques nouvelles des modalités, RPL 1937
- ▶ Directions nouvelles de la logistique aux E-U, RPL 1937
- ▶ Les Systèmes formalisés des modalités aristotéliennes, 1950.

## Robert Feys: publications – Books

- ▶ Logistiek : geformaliseerde logica, 1944
- ▶ De ontwikkeling van het logisch denken, with A. Borgers 1949
- ▶ Gentzen Recherches sur la déduction logique, with J. Ladrière, 1955
- ▶ Combinatory logic with H. B. Curry and W. Craig, 1958
- ▶ Modal logic with MacKinsey, completed J. Dopp, 1965
- ▶ Dictionary of symbols of mathematical logic with Fitch 1969

# Robert Feys: Teaching

In Brussels :

- ▶ Morning lecture (other): ordinary students, ordinary logic
- ▶ Evening lecture (Feys): better students, advanced logic, experimental logic, logic calculus as logical juggle!

In Leuven:

- ▶ Logistique (formalized logic), Mathematical logic
- ▶ Philosophy, Morale (ethics), metaphysics
- ▶ +/- 30 syllabi; +/- 20 unpublished notes etc.

Feys "*developed logics dual of intuitionistic systems*",  
"*jolly logic monsters which are consistent but in which one single important principle is not valid: the principle that forbid contradiction*" (from the 30's L& A, 1960)

# Robert Feys's Modal Logic

- ▶ Since the MA, logic taught was Aristotle's syllogistic
- ▶ 1880 Pope Leon XIII: Institute of Philosophy. Against positivism, St Thomas Aquinas. Sep 2024 KULeuven!
- ▶ Context: Brouwer, Heyting, Barzin, Errera, Glivenko,
- ▶ E. Beth, von Wright... later on
- ▶ 1948–1953 references to a book on ML by Feys-McKinsey

# Congress and Meetings

- ▶
- ▶ 1938 Paris
- ▶ 1948 Zurich -the idea of dialectics -see Beth, infra
- ▶ 1953 Brussels Modal logic past and future
- ▶ 1955 Paris CNRL
- ▶ 1957 ASL meeting, Cornell,  
Kripke 17 attended, got all of it and more. JSL 1959



## Feys, Perelman et alii: CNRL-NCNL + BSLPS



- ▶ Started 1950; Registered 1955; L&A, 2nd ser. 1958.
- ▶ People from State U. Ghent, Liege; Free U. Brussels; Catholic U. Leuven : Ph. Devaux, J. Dopp, Ch. Perelman, L. Apostel, J. Ladrière, ...
- ▶ Journal: "Logique et Analyse", 1st ser., (n°1) Nov 1954 (n°8) Feb 58.
- ▶ In de statutes, 2 orientations: Formal (Feys) Informal (Perelman)

## Arnould Bayart (Beselaere 1911–Brussels 1999) CV.



- ▶ 1941 Doctorate in the Law. Central Jury, KULeuven
- ▶ 1935 Lic. Philosophy; 1952 Doctorate in Philosophy. Idem KULeuven
- ▶ Teaches Logic, SJ Faculties, Antwerpen
- ▶ 1962 Lawyer at the High Court, Stafhouder Brussel
- ▶ OECD Judge of the European Nuclear Energy Tribunal

## Bayart's Some Publications in Logic

- ▶ (Carnap, Barcan, Kanger, Kripke et al.,)
- ▶ Bayart, L&A 6, Apr. 1956 "Suggestions for Modal logic"
- ▶ Bayart, L&A, 7, Nov 1956 "Necessity and Provability"
- ▶ Bayart, L&A, 8, Feb. 1957 "Generalized  $\times$  and Exp of Order types"
- ▶ Bayart, L&A 2ser, 1, 1958 Soundness of S5 Modal PL
- ▶ Bayart, L&A 2ser, 2, 1959 Completeness of S5 Modal PL

# Bayart's Semantics of Modal Logic

- ▶ Straightforward today. Start from Leibniz and as usual:
- ▶  $U$ , universe with 0 and 1;  $W$ , set of worlds;  $I$ , set of individuals
- ▶  $\Box$  is 1 in all  $w \in W$ ;  $\Diamond$  is 1 in one  $w \in W$
- ▶ propositional and predicate variables evaluation
- ▶ connectives,  $\forall np$ ,  $\exists np$   $\Box p$  and  $\Diamond p$ , idem
- ▶ Fml  $f \in U$ ,  $M$  in  $U$  valid iff for all sets  $V$  of values in  $U$ ,  $f = 1$  for  $UMV$
- ▶ Fml  $f$  realizable iff for one  $M \in U$ ,  $f$  is realizable for  $UM$
- ▶ System Consistent if all fmlae are 1; Complete if they are all deducible
- ▶ Proofs with Henkin

## Nual Belnap (1930-2024) and Relevant logic



- ▶ NSA computer programmer
- ▶ 1958, Brussel-Louvain with Feys who suggested his
- ▶ 1956 L&A 1ser, Feys : Ackermann Strenge Implikation
- ▶ Ackermann's systems  $\Pi$  and  $\Pi'$
- ▶ Basis of Anderson & Belnap system E of Entailment
- ▶ "A useful 4-valued logic"
- ▶ "How a computer should think"
- ▶ Some students: R. Meyer, M. Dunn, A. Urquhart...

## Robert K. Meyer (1935-2009)



- ▶ BA Theo. Princ. 56; JPN; Phd Phil. Pittsburgh
- ▶ Topics in Modal and many-valued logic, 1966
- ▶ Preferred system, logic R (K minus EFQ)
- ▶ Kripke Decision JSL 1959 and IDP
- ▶ K2U, The key to the universe
- ▶ Feys–Curry polymorphism

## Robert K. Meyer: K2U

- ▶ 1972 Routley–Meyer system  $B+$  with Kripke relational semantics.
- ▶ Correspondence CL (lambda) calculus in another form) and  $B+$
- ▶ Hence CL is the "Key to the Universe".
- ▶ 1983 Barendregt, Coppo, Dezani intersection theory of type.
- ▶  $B+$  equivalent to this theory, i.e. relevant entailments coincide with the subtyping relation.
- ▶ Practical application, Cduce designed to deal with XML
- ▶ Its set-theoretic notion of subtyping corresponds to the relevant entailments of  $B+$ .

# Memories are made of this...

## Souvenirs from Gent

Dickson  
34 years of Knipke

(1)

1. State the open decision problem of  $T \rightarrow$

Mariangioli's Favorite Counterexample (MFC)

Let  $W$  be the  $\mathcal{B}$ -theory defined by ALL  
formulas  $(B \rightarrow B \rightarrow C) \rightarrow B \rightarrow C$

Let  $x$  be  $\uparrow(p \rightarrow, p \rightarrow) \wedge (q \rightarrow, q \rightarrow r)$

Let  $y$  be  $\uparrow(p \vee q)$

Then  $r \in W_x y$

Proof:  $p \rightarrow (p \rightarrow) \rightarrow (p \rightarrow r) \in W$

$p \rightarrow (p \rightarrow) \in x \quad \rightarrow E$

$p \rightarrow r \in W_x \quad \rightarrow E$

$q \rightarrow r \in W_x$  like manner

$(p \rightarrow) \wedge (q \rightarrow) \in W_x \quad \wedge I$


BUT  $(p \rightarrow) \wedge (q \rightarrow) \in p \vee q \rightarrow \rightarrow \text{subex}$

SO  $p \vee q \rightarrow r \in W_x \quad \rightarrow E$ -closure

SO  $r \in W_x y \rightarrow E$

BUT there is NO WAY  $r \in x y y$ ,  
falsifying  $W$  reduction principle

$W_x y = x y y$



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## Robert K. Meyer: Relevant Arithmetic #

- ▶  $R^\#$  is Peano Arithmetic (PA) with 1st-order relevant R
- ▶  $R^\#$  is consistent
- ▶  $R^\#$  admits  $\gamma$  (i.e.  $A \rightarrow B$  is  $\neg A \vee B$ , i.e. detachment for material implication)
- ▶ every thm of PA is a thm of  $R^\#$

## Robert K. Meyer: Relevant Arithmetic ##

- ▶ Problem: Harvey Friedman provided a counterexample (1992)
- ▶ No problem: we have  $R^{##}$  where  $\gamma$  is ok
- ▶ universal generalization rule ( $\forall xAx$ ) replaced by the  $\omega$ -rule  
(From  $A_0, A_1, \dots$ , infer  $\forall xAx$ , i.e. a premise for each  $n \in \mathbb{N}$ ).
- ▶ another way out: Robinson Arithmetic, a subsystem of PA

## OFF Topic but for information

### Wiles and alii's proof of FLT (1995)

R. K. Meyer on FLT and back in Ghent

- ▶ considered ok but intricate; requires too hard results
- ▶ requires too big structures: Grothendieck's universes
- ▶ out of reach of PA and even ZFC?
- ▶ Is there an elementary proof?
- ▶ a proof relying on ad hoc hypotheses?
- ▶ a  $R^{##}$  proof immune to Gödel?
- ▶ ... ?

# Elementary proof of FLT!

- ▶ McIntyre's program: express MT, the Modularity Thesis, central to the proof as an arithmetic statement provable in PA
- ▶ from there, a proof of FLT without MT
- ▶ his clues: Wiles's completion of structures ( $N$  or  $Q$ ) uses analytic and topological structures ok in PA
- ▶ so, replace completion by finite approximations within PA
- ▶ Friedman conjecture: FLT provable in EFA, weaker than PA, or in some of its conservative extension
- ▶ This means, back to Poincare, to Peano and to Meyer in the Newton da Costa  $P=NP$  way.

Back to the MLE book.

Evert Beth (1909-1964)



- ▶ Zurich 1948 Piaget et al., "The idea of Dialectics"
- ▶ Strong reaction against "principles of eidetism"
- ▶ No need to reintroduce "dialectic" in scientific vocabulary
- ▶ nor in Philosophy. Conservatism is marching backward
- ▶ and dangerous.

*We must absolutely get rid of an outdated and worn out tradition and replace it by an all new orientation conditioned by the current and future requirements of scientific and social life"*

Which tradition?